MPACT

Math and Computational Thinking Through 3D Making

Teacher Notes: Grade 7 Module 2

Goals for the Module

Math

Understand empirical probability and theoretical probability and explain differences between them.

Determine probabilities through reasoning and empirical testing.

Convert within a given measurement system.

Use approximations and the formula for the volume of a cube to reason about side lengths and volume of a cube.

Spatial reasoning

Mentally rotate objects decide if they are the same, to envision a new position for them.

Correctly sketch interpret 2D drawings of 3D objects.

Physically and mentally rotate solid shapes to design game pieces.

Envision the designs they have drawn as shapes or holes in Tinkercad.

Computational Thinking (CT)

Recognize and use patterns when testing a die.

Decompose problems into smaller pieces, such as the way to represent numbers on a die.

Create and follow algorithms, including to find the volume of a cube or its side length.

Understand the components of 3D shapes by using nets (decomposition).

Determine if your die is fair (testing/retesting).

Materials

- Polyhedral dice
- Other dice you can find: for example, cubes with dots, cubes with numbers, dodecahedra, pyramid shaped dice.
- Paper, including graph paper and isometric grid paper, if possible
- Standards school supplies
- Whole class display and document camera

Timing

- Seven to eight 45-minute lessons
- Times may vary. Times given below are for phases of design (such as prototyping) and you can break those into lessons for what works best for you.

Introduction 10 min

Goals

Design and Making

Understand the idea of design requirements.

Materials

- Presentation slides
- Design process poster

Introduce the module

10 min

Have students read aloud the paragraph on dice.

Ask students what they know about dice and whether and when they have used them.

Show some examples of real dice.

Throughout the module, we refer to *die* as the singular and *dice* for the plural.

Read the paragraph ("In this module...") about the goal of the unit. Ask students to restate what they understood their task was in this unit.

- Probe for what it means to make a dice "fair." For example:
  - It cannot land on the same number all the time.
  - Each number should come up about the same number of times, if you roll it long enough.

Explain the **design process** (4-part model)

- Collect ideas.
- Make and remake prototypes (define prototype as a quick sketch or 3D model made with easy-to-fail materials that you can discard easily to try again.)
- Design on paper and computer—we will be using a computer-assisted design program (CAD) called Tinkercad™.
- Make the final dice.

Explain that designing requires iteration (the process of going back and fixing or changing your first and second and third tries). We will do it as much as possible, but we all have to work efficiently to make that happen. (This is because the pace of the unit is brisk.)

Collect Ideas 15 min

Goals

Design and Making

Brainstorm a requirements list.

Get information on user needs and wants and modify the requirements list.

Materials

- https://www.youtube.com/watch?v=s-FM8GM6bN4
- A YouTube video in which a teenager describes modifications available for the blind and visually impaired (included in the slide presentation)
- A variety of dice: cubes with dots, cubes with numbers, dodecahedra, pyramid shaped dice.
- Isometric grid paper
- Whole class display and document camera

Brainstorm requirements

15 min

Q1. Discuss a couple of requirements together as an example. Possible responses:

- It should have six faces.
- You should be able to tell the difference between the faces with numbers 1–6 with your fingers.
- It can't be too small or the different numbers won't be easily felt or seen.

Q2. In teams, students generate their own list of requirements. *Sample Answers:*

- *It should have bumps or holes / be smooth or rough.*
- *There is an equal chance rolling any number.*
- *Every face has to have equal weight.*
- *No two faces can be the same.*
- *Two faces can't have different weight.*
- *Each face has to have the same shape.*

After students work in teams, have them share some of the requirements they added. This list will be revised.

Maintain a list of requirements

Students should use this list from the first day.

- When they brainstorm
- Anytime they have a change / update to the requirements
- When they design in Tinkercad
- After they print to verify that the requirements have been satisfied

Make and Re-make Prototypes 65 min

Goals

Design and Making

Know what a prototype is.

Make more than one prototype to try out ideas.

Math

Make and use a net of a cube.

Spatial Reasoning

Imagine a 2D net folded into a 3D object.

Mentally rotate 3D objects represented in 2D.

Materials

- Scrap paper
- Nets printed from the student materials, if possible.

Make nets and prototypes for a die for the visually impaired 35 min

Q1. Students should make nets of a cube and use them as a prototype. They should draw the way they will make the faces so the visually impaired can use them. Since they are drawing on a 2D surface, they will have to decide how to represent bumps or holes.

Circulate and

- **Encourage students to prototype more than one design.**
- **Ask how they are representing raised or indented parts of their designs.**

Brain Teaser 15 min

Q2. **Circulate and**

- **Ask students to imagine folding the cube from the net with the black square in front.**
- **Ask them to identify the other shapes that would be showing.**
- **Have students make a copy of the net and cut it out to check their answers.**

Answer: D

This spatial reasoning item is from the free web resource (https://www.123test.com/spatial-reasoning-test/) and you can find more tasks there to try with your students.

Ask Your Mentor 15 min

Students should answer the questions briefly but with detail.

They may customize the question they ask of their mentor.

Design the Die 140 min +

Goals

Design and Making

Optional: Draw a die using isometric grid paper.

Use Tinkercad to design a die with modifications for the visually impaired.

Math

Decide measurements for dice, based on requirements.

Solve a problem using cubed lengths and cube roots of length.

Spatial reasoning

Visualize the six faces of a cube.

Envision an object from all sides.

Optional: Draw isometric drawings to represent 3D shapes.

Computational thinking

State, follow and debug an algorithm.

Materials

- Isometric grid paper
- Tinkercad
- Rulers
- *Shortcut Keys* handout

Optional: Make an isometric drawing of the cube

15 min

Optional: Show students how to make an isometric drawing of a cube – to show how to represent a 3D shape on a 2D plane.

Point out that in isometric drawings, the square faces of the cube are represented as parallelograms. But, to our eyes, the shape can appear as 3D cube.

Have students create their own isometric drawing of their designed dice.

Set maximum and minimum side lengths for the die

15 min

The maximum size for the die is set here. Ask students why that would be set.

Q1. Ask students about the minimum size for their die, which may already be in their requirements list.

Answer: The die has to be large enough to play with but small enough to make a reasonable print time.

Decide on the exact size of the die

10 min

Q2–3. Students should ensure their cube has side lengths that meet the requirements for the maximum volume of the cube and the requirement for the minimum size (which may be a side length.)

Tell students that Tinkercad's smallest measurement is millimeters (mm).

Ask them how many mm are in 1 cm. (10)

Circulate and help students figure out these key facts:

- **What do we know about the side lengths of a cube? The sides of a cube are all the same length, so length, width, and height should be the same.**
- **What are the side lengths of the smallest die you can make? Students will need to decide how small the die can be and still be usable as a die to roll and to put designs on.**
- **How can we find the volume of a cube? Can you write it as a formula? The volume of a cube: $V=s^3$.**
- **What is the largest side length the cube can have? The largest volume, 45 cm^3, does not have a whole number cube root. (No whole number cubed is 45). So the largest side length is greater than 3 cm because $3^3=27$, but less than 4 cm because $4^3=64$.**
- **Ask students to try different lengths (to the nearest tenth of a cm) to find the precise length they want for their cube. For example, $3.5^3=42.9$ cm^3 so 3.5 cm is a good choice.**
- **Have students sketch their die and label it with the right size side lengths that they have chosen.**

Create an algorithm for the teacher

15 min

These questions not only address CT and spatial reasoning, but also give students a chance to develop skills with Tinkercad. They design a shape by using a methodical approach (algorithmic thinking) and visually and physically rotating objects in Tinkercad.

Q4. Students follow the instructions given to create the shapes and view in the picture. Make sure they can see the little arrows.

Q5. Students should write down instructions in words, step by step (each step a new sentence).

Circulate and n otice instructions that are correct and incorrect.

- **Choose a students' instructions to follow (one that is not quite right) and ask students why these instructions did not work. Ask the class to suggest changes.**
- **Do as many group's instructions if you have time. Be sure to include one that is correct.**

Q6. Students should answer for one of the instruction sets that you tried, their own if possible.

Design on the computer

50+ min

Students use Tinkercad to design a die with special accommodations for the blind. Help students envision the designs they have drawn as bumps or holes in Tinkercad.

Have rulers and *Shortcut Keys* handout available for students.

Create the cube design in Tinkercad

40 min

Q7–8. Show students how to make shapes and holes with Tinkercad.

Circulate and encourage students to keep on track by

- **designing based on their drawing and requirements.**
- **thinking about measurements, that they are working in mm, and the size of die in their requirements list may be given in cm.**
- **thinking about how big the die appears on the screen vs. how big it will be when actually printed. A ruler will help.**

Reflect and Celebrate

10 min

Make it a celebration! Ask students each of the questions on the student handout. Also, you can ask:

- Were there places you got stuck when using the CAD?
- Was there a time when you realized you hadn't met a requirement?
- Did you have any other problems?
- What did you do to get unstuck/meet requirements/solve problems?

Emphasize that they learned about mental rotation, creating an algorithm (CT) and measurements in cm and mm. They solved a problem with decimals to find the size of their cube.

Make the Real Thing 130 min +

Goals

Math

Compare empirical and theoretical probability, for small numbers of choices (12 or fewer).

Observe patterns.

Understand the probability of a chance event is between 0 and 1.

Approximate the probability of a chance event occurring by collecting data.

Develop a probability model with outcome of rolling the dice.

Spatial reasoning

Imagine a net folded up. Rotate the result mentally.

Computational Thinking

Debug their Tinkercad design if the printed die does not come out as expected.

Materials

- Whole class display, rulers, polyhedral dice

Before the lesson

For partner check:

- Confirm that you have all the students' dice in your Tinkercad classroom.

For printing:

- Export the files to your desktop and then import these files into the printer software to check the length of time they will each take to print. This will allow you to set a schedule for printing.
- If there is time, you can print a smaller version of some of the spools to check if they will print properly and come out as students expected.

Note: Exploring probability can be done while waiting for prints to be done.

Get ready to print

15 min

Q1. Students should ensure their die meet the requirements before printing.

- Designs must be usable by the visually impaired and sighted.
- Cubes must be between 8 cm^3 and 45 cm^3 in volume.

If students make a die with "bumps," the length and the width of their die might not be the same, depending on how they make their bumps. Make sure they include the part sticking out from a cube in their measurements.

Combine dice on the print bed, to fit.

Get students involved in this process as possible.

Optional: Give students the dimensions of the print bed to help estimate how many die can fit on it.

While waiting for prints: Explore probability, Introduction

15 min

Read the text aloud with the class.

Elicit the meaning of the 1 and 6 in the probability fraction.

Ask students if the fraction would change if the number to roll changes.

Answer: No, because each number appears only once.

Find probabilities

15 min

Q2. *Answers: 1/6; 1/8; 1/12*

Circulate and:

- **Ask students why each fraction has numerator 1. (There is only one way to roll the given number.)**
- **Ask them how they determined the denominator. (The number of faces on the die, so numbers possible to roll changed.)**

Test a die to see if it is fair

15 min

Q3–4. Elicit that the number of rolls a face (number) should come up is 60 divided by the number of faces on that shape die.

Whole class discussion after rolls:

- Ask why the numbers in column A did not come out the same as in column B.

  Possible answers: Column B only tells us what should happen; Column A tells us what actually happens.

- Ask: Will more rolls, say 1000 rolls, lead to different results?

  Possible Answer: Yes, if the die is fair, the more rolls you make, the closer you get to the number of rolls that should occur.

  Students may have an intuitive feel for this, which is called the law of large numbers. They can test it by adding their tallies to another group's who used the same type of die.

Q4. *Possible Answers:*

- *Yes, because all the faces are the same size and there are no bumps or holes that would make one face more likely to come up. Or no, because the faces are not the same.*
- *No, because the actual number of rolls came out different from what was expected when a die was fair.*
- *Not all faces came up in equal numbers, but we are not sure if it is a problem with a die or if my test is not large enough.*
- *I did not roll with consistency, so I don't know.*
- *I want to roll the die more.*

Explore ideas about "law of large numbers." 10 min

Q5. Use the questions to lead a discussion.

Answers:

- *True. The greater the number of rolls, the more accurate the empirical probability will be.*
- *False. This exact match is unlikely to happen, but the two should get closer and closer.*
- *False. It could turn out this way, but it is unlikely to.*

Q6. **Have students check their requirements to see if the printed die meets them.** Students check off the requirements they met or did not meet. Have them debug their design by explaining what went wrong and how they would change it.

To decide if a visually impaired person could use the die, ask:

- **How does your die look and feel?**
- **If you close your eyes, can you tell the difference between the six faces?**
- **Was it the size you expected? Why or why not?**

Q7. Explain we are trying to find out if their die is fair, as we did before. If it is fair, the number of times a number *is* rolled will be close to the number of times it *should be* rolled. Ask students to work in pairs: one person rolls, the other records, trade after 30 rolls.

Students should use tallies to keep track the number of times each number on the die *was actually* rolled (was face up).

Circulate and ask:

- **Elicit from students, if possible: The number of times the number *should* be rolled can be found by multiplying the theoretical probability (as in the introduction) by the number of rolls. (Or, the number of times the die was rolled, divided by the number of faces).**
- **Ask: Will more rolls, say 1000 rolls, lead to different results?**

  Answer: Yes, if the die is fair, the more rolls you make, the closer you get to the number of rolls that should occur. This was explored above. They can test this by adding their tallies to another group's who used the same type of die.

Q8. Students may answer yes or no, depending on how close the number of times rolled got to "should be rolled."

Possible answer: it could be fair, but more rolls are needed.

This agrees with the "law of large numbers", which says basically that the more rolls, the closer "actual" gets to "should."

They may also see flaws in their design that make the die unfair. For example, the pieces that stick out to represent the numbers may weigh a lot more for 6 than for 1.

Q9. Answers will vary and can either address improving the die for the visually impaired or making the die fair.

When prints come back: test the die for the visually impaired 30 min

Reflect and Celebrate 10 min

Celebrate completing the unit! Make sure students feel accomplished.

Focus on the positive. Include spatial reasoning and CT learning too.

20 min

Ask Your Mentor

Help students communicate their design experience with mentors and learn about how professionals solve problems in the face of challenges.

20 min

Test Your Knowledge of Probability

Can be assigned as homework.

Q10. *Answers:*

a. *15/50 This is the theoretical probability*
b. *50/50 or 1. You will always get a defective marble. Notice this is NOT the same as saying the odds are 50-50. There the 50s are percents.*
c. *0. There is no way to get a defective marble.*
d. *No, it cannot. If the probability of something happens is 1, that means it will always happen.*

Q11. *Answers:*

Here are the pairs you could pick:

- *White, one of the black dice*
- *White, the other black die*
- *Both black.*

So, it is more likely that you will get one of each color (2/3) than two blacks (1/3). OR *It is more likely see that only one of them is black because there are three possible events, BW, BW, BB, and the probability of getting BW is 2/3 while the probability of getting BB is 1/3.*

15 min

Brain Teaser

Can be assigned as homework.

This problem came from the website (https://www.123test.com/spatial-reasoning-test/) that provides spatial reasoning tests for free.

Q12. *Answer: B*

Q13. *Possible answers:*

- *I imagined the cube with the shape in the front as the front part of the folded cube and imagined the folds to see what would be on the other faces.*
- *I sketched the cubes.*